$$\mathcal{L}^{-1}\left\{\underbrace{\frac{2s^2+3s+2}{s(s+1)^2}}\right\} = \mathcal{L}^{-1}\left\{\underbrace{\frac{\cancel{A}^{2}}{s} + \frac{\cancel{B}^{0}}{s+1} + \frac{\cancel{C}^{-1}}{(s+1)^2}}\right\}$$

MULTIPLY BY $s(s+1)^2$

$$2s^2+3s+2 = A(s+1)^2 + Bs(s+1) + Cs$$

$s=0$: $\quad 2 = A$

$s=-1$: $2-3+2 = C(-1) \longrightarrow 1 = -C \longrightarrow C = -1$

COEF OF s^2 : $\quad 2 = A+B \longrightarrow B = 2-A = 0$

MANDATORY SANITY CHECK : $s=-2 \quad \dfrac{8-6+2}{(-2)(-1)^2} = \dfrac{4}{-2} = -2$ ✓

$$\frac{2}{-2} + \frac{-1}{(-1)^2} = -1 + -1 = -2$$

$$\mathcal{L}^{-1}\left\{\frac{2}{s} - \frac{1}{(s+1)^2}\right\} = 2 - te^{-t}$$

$$\mathcal{L}^{-1}\left\{\frac{\quad}{(s+3)^3 s^4}\right\} = \mathcal{L}^{-1}\left\{\frac{A}{s+3} + \frac{B}{(s+3)^2} + \frac{C}{(s+3)^3} + \frac{D}{s} + \frac{E}{s^2} + \frac{F}{s^3} + \frac{G}{s^4}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{8s-2s^2-14}{\underbrace{(s+1)(s^2-2s+5)}}\right\} = \mathcal{L}^{-1}\left\{\frac{A}{s+1} + \frac{Bs+C}{s^2-2s+5}\right\}$$

DISCRIMINANT

b^2-4ac

$=(-2)^2-4(1)(5)<0$

IRREDUCIBLE

DIFFERENT VALUES OF B, C

$$= \mathcal{L}^{-1}\left\{\frac{\overset{-3}{\cancel{A}}}{s+1} + \frac{\overset{1}{\cancel{B}}(s-1)+\overset{1}{\cancel{C}}(2)}{(s-1)^2+4}\right\} = -3e^{-t} + e^{t}\cos 2t + e^{t}\sin 2t$$

$$8s-2s^2-14 = A\left((s-1)^2+4\right) + B(s-1)(s+1) + C(2)(s+1)$$

$s=-1$: $-8-2-14 = A(8) \longrightarrow -24 = 8A \longrightarrow A=-3$

$s=1$: $8-2-14 = A(4)+C(2)(2) \longrightarrow -8=-12+4C$

$C=1$

COEF OF s^2: $-2 = A+B \rightarrow -2=-3+B \rightarrow B=1$

MANDATORY SANITY CHECK: $s=2$ $\dfrac{16-8-14}{(3)(4-4+5)} = \dfrac{-6}{3\cdot 5} = \dfrac{-6}{15} = \dfrac{-2}{5}$ ✓

$$\frac{-3}{3} + \frac{1(1)+1(2)}{5} = -1+\frac{3}{5} = -\frac{2}{5}$$

~~$\dfrac{8s-2s^2-14}{(s+1)(s^2-2s+5)} = \dfrac{8s}{(s+1)(s^2-2s+5)} - \dfrac{2s^2}{(s+1)(s^2-2s+5)} - \dfrac{14}{(s+1)(s^2-2s+5)}$~~

SOLVE $y''+6y'+9y=0$, $y(0)=-1$, $y'(0)=6$

$$\mathcal{L}\{y''+6y'+9y\}=\mathcal{L}\{0\}=0 \qquad Y=\mathcal{L}\{y\}$$

$$s^2Y-sy(0)-y'(0)+6(sY-y(0))+9Y=0$$

$$s^2Y+s-6+6(sY+1)+9Y=0$$

$$(s^2+6s+9)Y=-s+6-6=-s$$

$$\mathcal{L}\{y\}=\frac{-s}{(s+3)^2}=\frac{A}{s+3}+\frac{B}{(s+3)^2}$$

$$y=$$